**Math Investigation Summer 2011**

**7x + 11y =100**

**The problem:**

Write 100 as the sum of two integers, one divisible by 7 and the other divisible by 11. Use  
your answer to find formulas giving all the solutions of the following equation where x and  
y are integers.

**The investigation:**

Okay, so first I worked with positive integers to see how many pairs I could find to satisfy the equation. The only pair in which BOTH values gave me positive integers was:

56 + 44 = 100

Written to satisfy the equation:

7(8) + 11(4) = 100

x = 8 and y = 4

If this is the only one I could find that both values were positive. I also realized that I could not have 2 negatives because it would not work when adding. So I decided to move on from there. I decided to increase the values of x to at least find 3 more pairs with a positive values for x. So what I did was to start from 56 and write more multiples of 7 in a table:

|  |  |  |  |
| --- | --- | --- | --- |
| 56 | 8 | 44 | 4 |
| 63 |  | 37 |  |
| 70 |  | 30 |  |
| 77 |  | 23 |  |
| 84 |  | 16 |  |
| 91 |  | 9 |  |
| 98 |  | 2 |  |
| 105 |  | -5 |  |
| 112 |  | -12 |  |
| 119 |  | -19 |  |
| 126 |  | -26 |  |
| 133 | 19 | -33 | -3 |
| 140 |  | -40 |  |
| 147 |  | -47 |  |
| 154 |  | -54 |  |
| 161 |  | -61 |  |
| 168 |  | -68 |  |
| 175 |  | -75 |  |
| 182 |  | -82 |  |
| 189 |  | -89 |  |
| 196 |  | -96 |  |
| 203 |  | -103 |  |
| 210 | 30 | -110 | -10 |
| 217 |  | -117 |  |
| 224 |  | -124 |  |
| 231 |  | -131 |  |
| 238 |  | -138 |  |
| 245 |  | -145 |  |
| 252 |  | -152 |  |
| 259 |  | -159 |  |
| 266 |  | -166 |  |
| 273 |  | -173 |  |
| 280 |  | -180 |  |
| 287 | 41 | -187 | -17 |

So from here, I got my 3 pairs including the original. The 3 new pairs are:

1. 133 + -33 = 100

7(19) + 11(-3) = 100

x = 19 y = -3

1. 210 + -110 = 100

7(30) + 11(-10) = 100

x = 30 y = -10

1. 287 + -187 = 100

7(41) + 11(-17) = 100

x = 41 y= -17

But I actually still had to go using negative integers for x which I believed would be giving me positive values for y. In other words, what I did was use the negative multiples of 7 next, starting from 1 x -7. This time I at least wanted to get 4 pairs. These are in the following table.

|  |  |  |  |
| --- | --- | --- | --- |
| -7 |  | 107 |  |
| -14 |  | 114 |  |
| -21 | -3 | 121 | 11 |
| -28 |  | 128 |  |
| -35 |  | 135 |  |
| -42 |  | 142 |  |
| -49 |  | 149 |  |
| -56 |  | 156 |  |
| -63 |  | 163 |  |
| -70 |  | 170 |  |
| -77 |  | 177 |  |
| -84 |  | 184 |  |
| -91 |  | 191 |  |
| -98 | -14 | 198 | 18 |
| -105 |  | 205 |  |
| -112 |  | 212 |  |
| -119 |  | 219 |  |
| -126 |  | 226 |  |
| -133 |  | 233 |  |
| -140 |  | 240 |  |
| -147 |  | 247 |  |
| -154 |  | 254 |  |
| -161 |  | 261 |  |
| -168 |  | 268 |  |
| -175 | -25 | 275 | 25 |
| -182 |  | 282 |  |
| -189 |  | 289 |  |
| -196 |  | 296 |  |
| -203 |  | 303 |  |
| -210 |  | 310 |  |
| -217 |  | 317 |  |
| -224 |  | 324 |  |
| -231 |  | 331 |  |
| -238 |  | 338 |  |
| -245 |  | 345 |  |
| -252 | -36 | 352 | 32 |
| -259 |  | 359 |  |
| -266 |  | 366 |  |
| -273 |  | 373 |  |
| -280 |  | 380 |  |
| -287 |  | 387 |  |

I found the following 4 pairs:

1. -21 + 121 = 100

7(-3) + 11(11) = 100

x = -3 y = 11

1. -98 + 198 = 100

7(-14) + 11(18) = 100

x = 30 y = -10

1. -175 + 275 = 100

7(-25) + 11(25) = 100

x = -25 y= 25

1. -252 + 352 = 100

7(-36) + 11(32) = 100

x = -36 y= 32

Now I have many values of x and y, positive and negative. Now I write them in order in the following table:

|  |  |
| --- | --- |
| **x** | **y** |
| -36 | 32 |
| -25 | 25 |
| -14 | 18 |
| -3 | 11 |
| 8 | 4 |
| 19 | -3 |
| 30 | -10 |
| 41 | -17 |

The problem here is how to use the sequence, whether you are looking for values that increase or decrease, positive or negative values. What I realized here was that all the numbers start to either be positive or negative after and before the pair x = 8 and y = 4. This pair really doesn’t have any negatives in it so I used it as the point zero of my values, the neutral point or the division line between positives and negatives of x, and the positives and negatives of y. This meaning, I’m not using it in any of my sequences as if it has no value, my division line. So the sequences would be the following (Note that they are with their pairs).

**Positive x: Negative y:**

19, 30, 41 -3,-10,-17

**Negative x: Positive y:**

-3, -14, -25, 36 11, 18, 25, 32

**WHEN x IS POSITIVE AND y IS NEGATIVE. x and -y.**

**Positive x:**

19 30 41

+11 +11

This means the sequence goes adding 11, meaning the sequence would start as 11n, saying you multiply the value of n by 11. n is the symbol used to stand as any number in a sequence, in this case 19 would give us n = 1 to say it is the 1st term of the sequence, 30 gives us n = 2 (2nd term) and so on to look like this.

|  |  |
| --- | --- |
| **n** | **value of x** |
| 1 | 19 |
| 2 | 30 |
| 3 | 41 |

Knowing that n is multiplied by 11 the table looks like this.

|  |  |  |
| --- | --- | --- |
| **n** | **n multiplied by 11** | **value of x** |
| 1 | 11 | 19 |
| 2 | 22 | 30 |
| 3 | 33 | 41 |

Now what I did was to look at the difference between the multiples of 11 to its respective value of x.

|  |  |  |  |
| --- | --- | --- | --- |
| **n** | **n multiplied by 11** | **value of x** | **Difference** |
| 1 | 11 | 19 | +8 |
| 2 | 22 | 30 | +8 |
| 3 | 33 | 41 | +8 |

Now this means the rule for positive x is: 11n + 8. This mean, the term you use, times 11 and then add 8. 11n+8 = positive x

**Negative y:**

-3 -10 -17

-7 -7

This shows the sequence goes subtracting 7, meaning the sequence would start as -7n, saying you multiply the value of n by -7. So in this case -3 would be the 1st term of the sequence, 30 the 2nd term and 17 the 3rd.

Doing the same thing as before with positive x, the table would be as following.

|  |  |  |  |
| --- | --- | --- | --- |
| **n** | **n multiplied by -7** | **value of y** | **Difference** |
| 1 | -7 | -3 | +4 |
| 2 | -14 | -10 | +4 |
| 3 | -21 | -17 | +4 |

This means that we are multiplying the terms by is -7 which gives us -7n. Now after multiplying the term the difference said to me that I had to add 4. So the rule for negative y is: -7n + 4. -7n + 4 = negative y

***So now that I had both rules I put them together to satisfy the equation:***

7x + 11y = 100

Now I replaced the x and y with the result was like this:

7(11n + 8) + 11(-7n + 4) = 100

**NOTE\* THE TESTING OF THE RULES ARE AT THE END OF THE INVESTIGATION BEFORE THE CONCLUSION.**

CONTINUATION

**WHEN x IS NEGATIVE AND y IS POSITIVE. -x and y.**

**Negative x:**

-3 -14 -25 -36

-11 -11 -11

This shows the sequence goes subtracting 11, meaning the sequence would start as -11n, saying you multiply the value of n by 11. So in this case -3 would be the 1st term of the sequence, -14 the 2nd term, -25 the 3rd and -36 the 4th term.

This would be the table, showing the term multiplying by -11 and the difference with the value of x.

|  |  |  |  |
| --- | --- | --- | --- |
| **n** | **n multiplied by -11** | **value of x** | **Difference** |
| 1 | -11 | -3 | +8 |
| 2 | -22 | -14 | +8 |
| 3 | -33 | -25 | +8 |
| 4 | -44 | -36 | +8 |

This means that we are multiplying the terms by is -11 which gives us -11n. Now after multiplying the term the difference said to me that I had to add 8. So the rule for negative y is: -11n + 8. -11n + 8 = negative x

**Positive y:**

11 18 25 32

+7 +7 +7

So this shows the sequence goes adding 7, meaning the sequence would start as 7n, saying you multiply the value of n by 7. So in this case 11 would be the 1st term of the sequence, 18 the 2nd term, 25 the 3rd and 32 the 4th term.

|  |  |  |  |
| --- | --- | --- | --- |
| **n** | **n multiplied by 7** | **value of y** | **Difference** |
| 1 | 7 | 11 | 4 |
| 2 | 14 | 18 | 4 |
| 3 | 21 | 25 | 4 |
| 4 | 28 | 32 | 4 |

This means that we are multiplying the terms by is 7, which gives us 7n. Now after multiplying the term the difference said to me that I had to add 4. So the rule for negative y is: 7n + 4. 7n + 4 = positive y

***So now that I had both rules I put them together to satisfy the equation:***

7x + 11y = 100

Now I replaced the x and y with the result was like this:

7(-11n + 8) + 11(7n + 4.) = 100

**TESTING THE RULES:**

***7x + 11y = 100***

***When x is positive and y is negative:***

7(11n + 8) + 11(-7n + 4) = 100

***When x is negative and y is positive:***

7(-11n + 8) + 11(7n + 4.) = 100

**The following are the rules tested using the 100th term to see if they worked:**

*When x is positive and y is negative:*

|  |
| --- |
|  |
| 7(11n+8) +11 (-7n+4) = 100 |
| 7(11[100]+8) + 11 (-7[100]+4) =100 |
| 7(1100+8) + 11(-700+4) = 100 |
| 7(1108) + 11(-696) = 100 |
| 7756 +- 7656 = 100 |
| 7756 - 7656 = 100 |

*When x is negative and y is positive:*

|  |
| --- |
|  |
| 7(-11n +8) + 11(7n+4) = 100 |
| 7(-11[100]+8) + 11 (7([100]+4) =100 |
| 7(-1100+8) + 11(700+4) = 100 |
| 7(-1092) + 11(704) = 100  -7644 + 7744= 100 |

What I did here in both, was substitute n with 100 then work out the rest of the rule.

By doing this I tested the rules and found out they work.

**CONCLUSION**

In conclusion, the solution to find the values of x and y HAD to be started using the pair x= 8 and y= 4 as the line of division between the negative and positive values as if it were number 0 in a number line, where it has no value as a negative nor a positive, in this case the pair had no negatives meaning it would not fit with any of the rules that were found. This lead to a result of find 2 different rules to find more values of x, and to find more values of y which satisfied the equation 7x + 11y =100.